Experimental Analysis of Frequency Transfer Uncertainty

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***Abstract* — An experimental analysis of frequency transfer uncertainty is carried out. In situations where at least two transfer techniques are available a first difference statistic is used to determine the type and level of the transfer noise for transfer methods such as common-view GPS, carrier-phase GPS, and two-way time transfer. Frequency transfer uncertainties approaching 1×10^{-16} at 30 days are possible. A method for estimating the frequency transfer uncertainty in situations where only one transfer technique is available is also examined.**

I. INTRODUCTION

Any time two frequency standards are compared over long distances some additional uncertainty is introduced into the comparison by the noise of the transfer system(s). Currently the most often used long distance transfer systems are GPS common view, GPS carrier phase, and Two-Way Satellite Time and Frequency Transfer (TWSTFT). However, increasing stress is being imposed on the transfer systems as the stability and accuracy of new standards such as cesium fountains and optical frequency standards improve. The frequency uncertainty introduced by a transfer system decreases with the time over which the frequency comparison is made, but there are practical limits to how long a standard can be operated in a more or less continuous fashion. Currently, the longest official comparison of a cesium fountain primary frequency standard to TAI is 60 days.

In this paper we examine the experimental evidence for the type and level of the instabilities in several time transfer techniques. In particular it is important to know whether the noise is white phase modulation (WPM) or flicker phase modulation (FPM). To help in this process we will use a first difference statistic initially discussed in [1]. In addition, when one examines the literature there is very little information on how to calculate the frequency transfer uncertainty (FTU) for the various transfer noise types. An accompanying paper in this proceedings "A Theoretical Analysis of Frequency Uncertainty" by Gianna Panfilo and Thomas E. Parker addresses this issue [2] and shows that the Allan deviation is not always an accurate measure of FTU. This paper provides a firm theoretical evaluation of the frequency transfer uncertainty as determined experimentally with the first difference statistic of [1]. For noise types where the mean frequency is zero (WPM, FPM, and random walk PM (RWPM), which is equivalent to white frequency modulation (WFM)), the first difference statistic of [1] is equivalent to the fractional frequency uncertainty calculated in [2].

II. CALCULATING FREQUENCY TRANSFER UNCERTAINTY WHEN TWO INDEPENDENT TRANSFER TECHNIQUES ARE AVAILABLE

When two (hopefully independent) transfer techniques are available between the same two frequency sources one can difference the two time series (one from each transfer technique). This results in a new time series that eliminates the clock noises and frequency offset and just contains the combined noise of the two transfer techniques. This is a very useful tool (referred to here as a double difference) in helping to determine the noise level and noise type of time and frequency transfer instabilities.

A. Experimental Observation of FTU for Different Transfer Techniques

Figure 1 shows the time deviation (TDEV) of UTC(NIST)-UTC(USNO) for two different paths over a two year period in 2005 and 2006. (UTC is Coordinated Universal Time.) One is a direct, relatively short base line, GPS common-view (CV) link using multi-channel receivers and the International GNSS Service (IGS) ionosphere models. The other is an indirect two-way (TW) link via Physikalisch-Technische Bundesanstalt, PTB. The two-way links from the National Institute of Standards and Technology, NIST, to PTB and PTB to the United States Naval Observatory, USNO, are both at Ku-band using the communications satellite Intelsat 707. There is currently no direct two-way link between NIST and USNO. All data are for 1 day averages. Since the clocks are maser ensembles at both ends (and hence very quiet) the TDEVs at τ less than about 3 days for both the CV and TW links show transfer noise that is FPM in nature at a level of about 300 ps. At larger τ values the TDEV shows clock noise. The CV and TWSTFT curves are not identical because both time series have some missing data. The decrease in the TDEV for the CV and TWSTFT data for τ larger than 100 days occurs because both UTC(NIST) and UTC(USNO) are steered to UTC. When the time series for TWSTFT and common-view are differenced (now a double difference) the long-term clock noise drops out and TDEV is much lower at τ greater than 5 days. The TW-CV TDEV curve represents the combined noise of two-way and common view (assuming two-way and common view are independent and largely uncorrelated) and it is roughly FPM in nature at a level of about 400 ps essentially for all τ values. The small bump near 150 days in the TW-CV curve is probably an indication of an annual cycle in the time delay of one or both transfer systems.

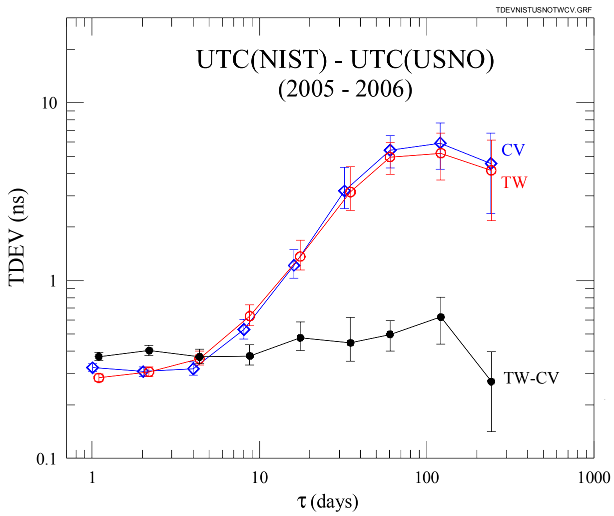

Figure 1. Time deviation plots of UTC(NIST)-UTC(USNO) as observed with GPS common view (diamonds) and TWSTFT via PTB (circles). The solid dots are for TWSTFT minus common view.

The first difference statistic of [1] can be used to calculate the frequency transfer uncertainty from the double differenced time series data used to calculate the TW-CV TDEV plot. This statistic, called $\sigma_{ft}(A,\tau)$ here, is defined by Eq. 1, and its calculation is illustrated in Fig. 2:

$$\sigma_{ft}^2(A,\tau) = \frac{\left\langle (\bar{x}_{t+\tau} - \bar{x}_t)^2 \right\rangle}{\tau^2} = \frac{1}{\tau^2 n}\sum_{i=1}^{n}\left(\bar{x}_{i+\tau} - \bar{x}_i\right)^2 \quad , \qquad (1)$$

where x_i bar is the average phase over interval A at epoch i, and τ is the interval between epoch i and i+τ. $\sigma_{ft}(A,\tau)$ is just the RMS frequency of the time series at interval τ. This statistic can be used in a meaningful way only in situations where there is no clock frequency offset or clock noise. An Allan deviation calculation on the same time series would be biased approximately 20 % high for WPM and FPM [2].

$\sigma_{ft}(A,\tau)$ is the frequency transfer error. Any value of $(x_{i+\tau} - x_i)/\tau = y_i$ that is not zero constitutes a frequency error introduced by the transfer systems. In the absence of known biases (a bias would be a nonzero average slope in the time series, or equivalently a nonzero mean frequency), the frequency transfer error is the frequency transfer uncertainty, FTU. In principle, a known bias could be measured and corrected for, in which case the FTU would be the RMS deviation about the bias (this would be equivalent to u_y in [2]). An example of a bias might be part of an annual cycle in transfer delay that could have a nearly linear component over an interval of several months. Another example might be an aging mechanism in one of the components of a transfer system that could look like a linear (or nearly linear) change in delay over a period of time. If biases are present that are poorly understood (and hence uncorrectable) then the frequency transfer error of $\sigma_{ft}(A,\tau)$ should be considered the frequency transfer uncertainty.

Figure 3 shows the combined FTU as calculated from $\sigma_{ft}(A=1d,\tau)$ for a NIST-PTB link using the double difference of two-way (TW) minus common view (CV), over the two year interval (730

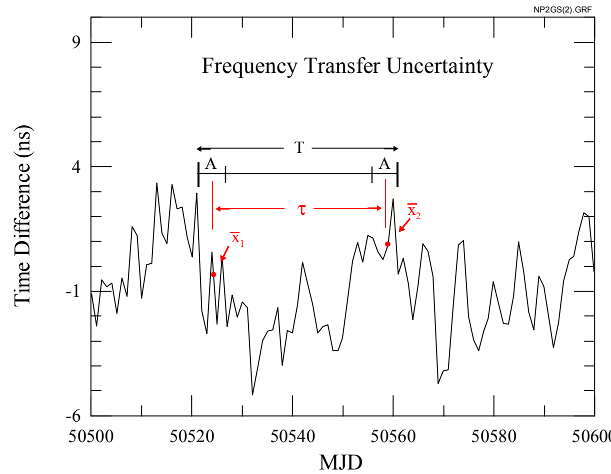

Figure 2. Illustration of how $\sigma_{ft}(A,\tau)$ is calculated. x bar is the average phase over the interval A.

days) covering the years 2005 and 2006 (upper (blue) curve). From TDEV data (not shown) it is clear that the FTU in Fig. 3 for NIST-PTB is dominated at small τ by the noise in common-view. The value of TDEV at 1 day for TWSTFT between NIST and PTB is about 150 ps (with masers at both ends) while for common view the TDEV at 1 day is about 500 ps. The NIST-USNO plot (middle (red) curve) is for two-way minus common view and comes from the same time series data as used for Fig. 1. Again this is for the two year period of 2005 and 2006. $\sigma_{ft}(A=1d,\tau)$ is about 30 % lower here than for the NIST - PTB link. The lower noise for NIST-USNO is primarily because of the better (shorter) common-view link. The NIST-CH plot is for two-way minus carrier phase (CP) between NIST and the Swiss Federal Office of Metrology, METAS, and is from data supplied by Christine Hackman [3]. The NIST-CH data is for an interval of only 184 days in 2006. This link exhibits the lowest noise mainly because carrier phase is more stable than code based common view. Note that the slopes for the three curves are all nearly the same, reflecting the fact that all the link instabilities are close to FPM in nature. Though it is not obvious from the curves in Fig. 3, a

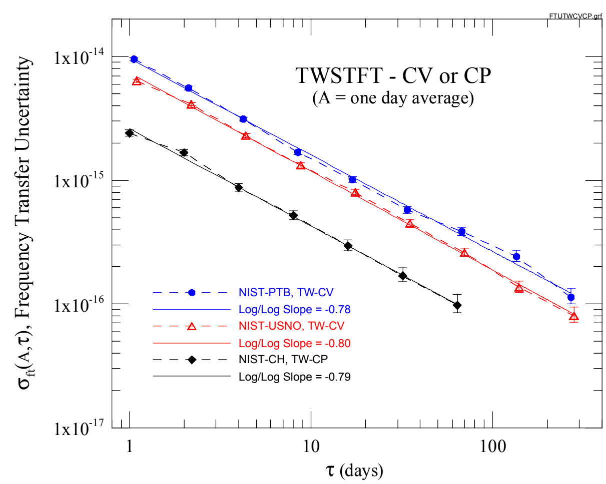

Figure 3. Examples of $\sigma_{ft}(A,\tau)$, FTU, as a function of τ for several different frequency transfer methods over three different links. The transfer methods are common-view GPS (CV), carrier-phase GPS (CP), and two-way (TW).

$\sigma_{ft}(A,\tau)$ curve for FPM noise is not a straight line on a log/log plot. This will be discussed in more detail in Section III B.

The TWSTFT links NIST-PTB and NIST-CH are very similar, and therefore the lower (black) curve in Fig. 3 would suggest that the upper (blue) curve is dominated by common view at all values of τ. Therefore one can conclude that the FTU for transatlantic common view is about 1×10^{-14} at 1 day and 7×10^{-16} at 30 days. The τ dependence is about $\tau^{-0.78}$, indicating the instabilities are FPM in nature. The FTU for the combined two-way and common view transfer techniques in the NIST-USNO link is about 30 % smaller and has a similar dependence on τ. However, since CV and TWSTFT have similar levels, it is difficult to draw any definite conclusions about the individual techniques. The combined noise of two-way and GPS carrier phase gives the lowest FTU of about 2.5×10^{-15} at 1 day and just less than 2×10^{-16} at 30 days. The τ dependence is about $\tau^{-0.79}$, which is similar to that of the other two curves. Some other two-way and GPS carrier phase links have shown a somewhat less steep τ dependence [3]. TDEV values at 1 day for the CP and TWSTFT data in the lower curve are nearly the same (with CP being slightly smaller), again making it difficult to draw any definite conclusions about the individual techniques. However, as will be shown in Section II C, it is not necessary to fully characterize the individual techniques in order to make definitive statements about a comparison uncertainty. Under the best of circumstances, using currently available frequency transfer techniques, it would take well over 300 days to reach FTUs approaching 1×10^{-17}.

The method for calculating the confidence limits shown in Fig. 3 is discussed in the following section.

B. Confidence Limits of $\sigma_{ft}(A,\tau)$

Here we will consider the confidence intervals for $\sigma_{ft}(A,\tau)$. To calculate the confidence intervals we will follow the method presented for the Allan deviation in [4-6]. In particular we know that for the Allan variance [4] the ratio

$$U=\frac{\hat{\sigma}_{ft}^2(A,\tau)}{\sigma_{ft}^2(A,\tau)}\nu \tag{2}$$

has a chi square distribution with ν degrees of freedom, where $\hat{\sigma}_{ft}^2(A,\tau)$ is the estimator related to the statistic $\sigma_{ft}^2(A,\tau)$. Based on the properties of the chi square distribution and on $E\left(\hat{\sigma}_{ft}^2(A,\tau)\right)=\sigma_{ft}^2(A,\tau)$ we can estimate the degrees of freedom using the relation:

$$\nu=\frac{2\left(\sigma_{ft}^2(A,\tau)\right)^2}{Var\left(\hat{\sigma}_{ft}^2(A,\tau)\right)} \tag{3}$$

After calculating the degrees of freedom we can obtain the confidence intervals for (2) using:

$$\frac{\nu}{b}\hat{\sigma}_{ft}^2(A,\tau)<\sigma_{ft}^2(A,\tau)<\frac{\nu}{a}\hat{\sigma}_{ft}^2(A,\tau) \tag{4}$$

where a and b are the percentiles of the chi square distribution at the confidence level considered p (usually the confidence levels are 68 %, 95 % and 99 %). The problem is to calculate the mean and the variance of the first difference statistic (1) in the case of white phase noise, flicker phase noise, and white frequency noise. Following the method reported in [6] and we have the following relation:

$$Var\left(\hat{\sigma}_{ft}^2(A,\tau)\right)=\frac{2}{M}\left(\sigma_{ft}^2(A,\tau)\right)^2\left(1+\frac{2}{M}\sum_{k=1}^{M-1}(M-k)\rho_k^2\right) \tag{5}$$

where $k=j-i>0$ and the correlation coefficient $\rho_{i,j}$ considering $Z_i=\dfrac{X_{i+\tau}-X_i}{\tau}$ is defined by:

$$\rho_{i,j}=\frac{E(Z_iZ_j)}{E(Z_i^2)} \tag{6}$$

Considering relation (5) and that in the case of (2) $M=N-\tau$, the degrees of freedom following (3) are:

$$\nu=\frac{N-\tau}{1+\dfrac{2}{N-\tau}\displaystyle\sum_{k=1}^{N-\tau-1}(N-\tau-k)\rho_k^2} \tag{7}$$

Following the method reported in [7] it is possible to obtain the expression for the correlation coefficients $\rho_{i,j}$. Here we report only the final values for the degrees of freedom for the white phase noise, white frequency noise, and flicker phase noise:

1. white phase noise: $$\nu=\frac{2(N-\tau)^2}{3N-4\tau} \tag{8}$$

2. white frequency noise: $$\nu=\frac{6(N-\tau)^2\tau}{2N-\tau+4N\tau^2-5\tau^3} \tag{9}$$

3. flicker phase noise: (10)

$$\nu=\frac{N-\tau}{1+\dfrac{2}{N-\tau}\left(\displaystyle\sum_{k=1}^{\tau-1}(N-\tau-k)\rho_{k,1}^2+\sum_{k=\tau+1}^{N-\tau-1}(N-\tau-k)\rho_{k,2}^2+(N-2\tau)\rho_{k,3}^2\right)}$$

where the correlation coefficients are given by the following relations:

$$\rho_{k,1}=\frac{-2\log(k)+\log(\tau^2-k^2)+2Ci(\omega_n\tau)-Ci(\omega_n(k+\tau))-Ci(\omega_n(\tau-k))}{2\gamma+2\log(\omega_n\tau)-2Ci(\omega_n\tau)} \quad if \quad k<\tau$$

$$\rho_{k,2}=\frac{-2\log(k)+\log(k^2-\tau^2)+2Ci(\omega_n\tau)-Ci(\omega_n(k+\tau))-Ci(\omega_n(k-\tau))}{2\gamma+2\log(\omega_n\tau)-2Ci(\omega_n\tau)} \quad if \quad k>\tau$$

$$\rho_{k,3}=\frac{-\gamma-\log(\omega\tau)+\log(2)+2Ci(\omega_n\tau)-Ci(2\omega_n\tau)}{2\gamma+2\log(\omega_n\tau)-2Ci(\omega_n\tau)} \quad if \quad k=\tau \tag{11}$$

and Ci(x) is the Cosine integral function, γ is Euler's constant, and ω_n is the bandwidth. This parameter is linked to the sampling theorem and can be obtained using the expression of the Allan variance in the case of flicker phase noise reported in [5]. To simplify the treatment the relations (11) can be used without the Cosine integral functions.

There are several advantages to using $\sigma_{ft}(A,\tau)$ rather than ADEV to calculate FTU. First of all $\sigma_{ft}(A,\tau)$ is unbiased for transfer noises, whereas ADEV is approximately 20 % too large for WPM and FPM noise, and is correct only for WFM noise. Also, the confidence limits are better for $\sigma_{ft}(A,\tau)$ than for ADEV. In addition, $\sigma_{ft}(A,\tau)$, being a first difference statistic, will be sensitive to slow time delay changes in the transfer systems that look like a frequency offset, but are real errors. ADEV, being a second deference, will not see these errors. The main disadvantage with $\sigma_{ft}(A,\tau)$ is that it cannot be used with a time series between two clocks where clock noise and a real frequency offset are present.

C. Combined Frequency Transfer Uncertainty for Two Independent Transfer Methods

If the noise processes in the two transfer techniques are independent (uncorrelated), the uncertainty of an unweighted average of frequency differences obtained from each transfer technique individually would be one half of the calculated $\sigma_{ft}(A,\tau)$ at the appropriate τ interval. For NIST-CH with GPS carrier phase and two-way used independently this gives an FTU of for the unweighted average about $1x10^{-16}$ at 30 days. To demonstrate that is true consider the example of NIST-CH where we will assume that we have two *independent* measures of the fractional frequency difference between two remote masers over an interval τ using two-way and carrier-phase GPS.

Let y_1 be the fractional frequency difference measured for the two masers via two way, and let y_2 be the measurement via GPS carrier phase. These are determined from the two time series of phase differences between the masers. If we do a straight unweighted average of the two measurements (this may not be optimal, but it is all we can do) we get

$$y_{Av} = \frac{y_1 + y_2}{2} = \frac{y_1}{2} + \frac{y_2}{2} \quad . \tag{12}$$

For each y_i measurement there is an associated uncertainty u_i. We don't know the values of the individual u_i's, but we do know the combined uncertainty, $u_c = \text{sqrt}(u_1{}^2+u_2{}^2)$. This is just $\sigma_{ft}(A,\tau)$ calculated from the time series generated by taking the difference between the GPS carrier phase and two-way time series as in Fig. 3. Thus $u_c = \sigma_{ft}(A,\tau)$.

What we want to calculate is u_{Av}, the uncertainty of y_{Av} for the interval τ. It is the square root of the quadrature sum of $u_i/2$ (similar to Eq. 12 but now dealing with random fluctuations).

$$u_{Av}^2 = \left(\frac{u_1}{2}\right)^2 + \left(\frac{u_2}{2}\right)^2 = \frac{\left(u_1^2 + u_2^2\right)}{4} = \frac{u_c^2}{4} \tag{13}$$

Thus

$$u_{Av} = \frac{u_c}{2} . \tag{14}$$

Therefore, the uncertainty of y_{Av}, u_{Av} (or FTU), is just $\sigma_{ft}(A,\tau)/2$, if the two techniques are uncorrelated. Unfortunately, this is probably not totally true. The degree of correlation between two-way and GPS carrier phase or GPS common view is not currently known, but it is very likely that there is some correlation with regard to environmental parameters with daily or annual cycles. Consequently, the degree of correlation is probably dependent on the value of τ. A very conservative approach would be to not use the factor of 2 in Eq. 14 and let $u_{AV} = u_c = \text{FTU}$. A more moderate approach would be to use $u_{AV} = u_c/\sqrt{2}$. The independence of different frequency transfer techniques is obviously an area for future investigation.

III. Estimating Frequency Transfer Uncertainty When Only One Transfer Technique Is Availible

If only one transfer path is available the task of determining the frequency transfer uncertainty becomes more difficult. An example of this is reporting the results of a Cs fountain primary frequency standard into TAI (International Atomic Time). TAI is a 'paper' time scale and does not physically exist in a single location. Though many different transfer techniques are used to transfer clock data through a complex network for use in TAI, there is, in effect, only one transfer path linking any particular lab to TAI. In this type of situation the transfer noise level and noise type must be estimated because they are largely obscured by clock noise. In some cases the clock noise is sufficiently small at short times that TDEV can be used to estimate the transfer noise at small τ values, as in Fig. 1. However, this does not give much information about the noise level and noise type at longer averaging times. Closure measurements provide some information, but they do not identify the noise characteristics between a specific pair of stations, and they may not include certain site-dependent instabilities [8].

A. Frequency Transfer into TAI

Recently the Bureau International des Poids et Mesures, BIPM, began publishing in Circular T the type A uncertainties, $u_A(k)_i$, of UTC-UTC(k) for each station k reporting clock data into TAI [8]. This prompted the Consultative Committee for Time and Frequency (CCTF) Working Group on Primary Frequency Standards to reexamine the expression used to calculate the frequency transfer uncertainty of a primary frequency standard reporting into TAI. Another motivating factor was evidence that time transfer instabilities had been significantly reduced over the past few years. Figure 4 shows TDEV plots from a comparison between the post-processed maser ensemble AT1E at NIST and TAI for two different periods. The upper (blue) curve with dots covers a 1.5 year period from November 1999 to May 2001. The TDEV values for τ in the range of 5 to 20 days represent transfer noise because they are too high to be clock noise, and the noise type is not white or flicker FM, as would be expected for clock noise. The lower (red) curve with triangles also shows TDEV for a 1.5 year interval around 2006. Over the 6 year period from 2000 to 2006 the transfer noise at $\tau = 5$ days has been reduced by about a factor of 3 through the increased use of multi-channel GPS common-view receivers, IGS measured ionosphere delay corrections, P3 (a two frequency, P code technique), and improved TWSTFT. The TDEV values at $\tau = 5$ and 10 days for the lower curve still represent time transfer noise. Though clock noise makes it difficult to identify the transfer noise type, the fact that TDEV decreases between $\tau = 5$ and $\tau = 10$ days indicates that there is some WPM noise present. In contrast, the data of Figs. 1 and 3 would suggest that instabilities in the most common transfer

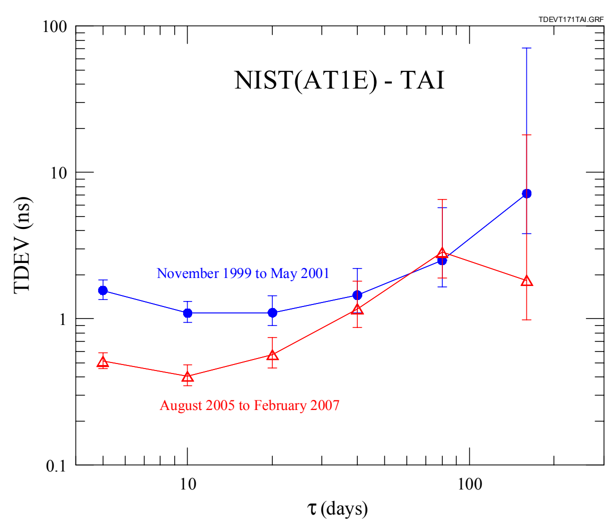

Figure 4. Time deviation for a comparison between the maser ensemble, AT1E, at NIST and TAI over an 18 month period around the year 2000, and a similar period around the year 2006. The decrease in TDEV at small values of τ is due improved time/frequency transfer.

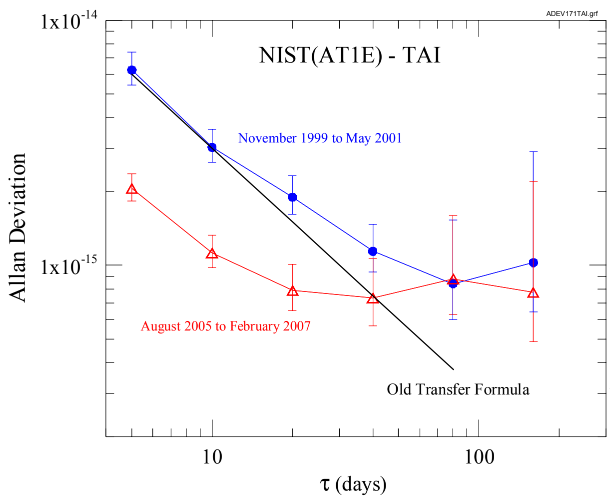

Figure 5. Allan deviation for a comparison between the maser ensemble, AT1E, at NIST and TAI over an 18 month period around the year 2000, and a similar period around the year 2006. The decrease in ADEV at small values of τ is due improved time/frequency transfer. The straight (black) line with no data points represents the frequency transfer uncertainty from the old formula used by the BIPM

techniques are mostly FPM, even beyond 100 hundred days. Transfer noise into TAI is obviously a unique situation because of the complex way TAI is calculated, and the large number of stations and great variety of equipment involved [8]. Therefore, it is not surprising that the noise characteristics might be different. This clearly is an area that needs further investigation.

Figure 5 shows Allan deviation plots from the same time series data that were used for Fig. 4. The improvement by a factor of 3 at small τ values is also clear here. The straight black line illustrates the old formula used to calculate the frequency transfer uncertainty, $u_{l/TAI}$, for primary frequency standards. The Allan deviation must be used here to estimate FTU because this is data between two clocks. It is a reasonable, but somewhat biased, estimate of FTU at small values of τ where transfer noise dominates over clock noise [2]. As can be seen in Fig. 5, the old formula, based on single-channel GPS common-view time transfer, was a good estimate of FTU around the year 2000. However, by 2006 it is clear that a new expression for frequency transfer uncertainty was needed. Furthermore, the values of $u_A(k)$ now published in Circular T are available. These values are estimated for each lab and eliminate a problem with the old formula in that the same expression was used for all labs reporting primary standards. Differing $u_A(k)$ values make it clear that the transfer uncertainty is not always the same for each lab (a number of different transfer techniques are in use).

B. New Frequency Transfer Equation

Equation 15 is the expression used by the BIPM until September 2006 to calculate the fractional frequency transfer uncertainty, $u_{l/TAI}$, introduced by the time transfer process when reporting a primary frequency standard measurement to TAI (black line in Fig.5). (In other words $u_{l/TAI}$ = FTU.)

$$u_{l/TAI} = 3x10^{-14}/\tau \qquad (15)$$

The same expression was used for all labs and the 1/τ dependence is that expected for WPM noise. In September 2006 a new expression was adopted at the recommendation of the CCTF Working Group on Primary Frequency Standards. This expression is shown in Eq. 16.

$$u_{l/TAI} = \left(\frac{\sqrt{u_A(k)_1^2 + u_A(k)_2^2}}{\tau_0} \right) \Big/ \left(\frac{\tau}{\tau_o} \right)^x \qquad (16)$$

Here $u_A(k)_i$ is the type A uncertainty (in seconds) of UTC-UTC(k) for station k at epoch i as reported in Circular T. τ_0 = 432x103 seconds (5 days) and is the data interval of UTC-UTC(k) in Circular T. $\tau = t_2 - t_1$ and is the report interval for the primary frequency standard. The value of the exponent x is currently 0.9. A value of x less than 1 was chosen to more accurately reflect the fact that there is very likely a significant component of FPM noise in the time transfer instabilities.

TDEV at τ_0 from two low noise clocks can be used to estimate the $u_A(k)_i$, and this can be used in an expression like Eq. 16 to estimate the frequency transfer uncertainty. For WPM noise, Eq. 16 using $u_A(k)_i$ = TDEV at τ_0, and x = 1 will give an exact value for the FTU. The situation is more complicated for FPM, as illustrated in Figure 6. A time series of simulated FPM noise was generated with TDEV at τ_0 = 1 day equal to 0.24 ns. This value was used for $u_A(k)_1$ and $u_A(k)_2$ in Eq. 16 with x = 0.9. The FTU using $\sigma_{ft}(A=1d,\tau)$ was calculated for the time series and is shown as the black dots in Fig. 6. The straight (red) line is a best fit to this data with a log/log slope of -0.875. A careful inspection of the dotted curve shows that the slope is not constant. At small τ the slope is approximately -0.7 and increases to about -1 at large τ. The blue line with diamonds shows the result from Eq. 16. The overall slope from the equation is in good agreement with the fit to the dotted curve, but the FTU level from the equation is biased about 25 % low. The bias between the equation and the data point at τ_0 = 1 day is only about 15 %, but the slope here is smaller than 0.9. Therefore, using TDEV in an expression such as that in Eq. 16 to estimate frequency transfer uncertainty for FPM noise is relatively complicated. The precise bias and exponent will depend on what range of τ (relative to τ_0) one is interested in.

Equation 16 as currently used to estimate the FTU for primary frequency standards should be considered a work in progress and may very well have to be modified in the future. These modifications

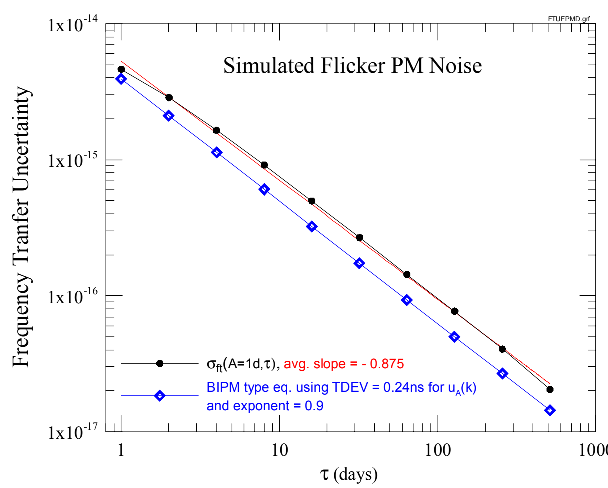

Figure 6. Comparison of frequency transfer uncertainty calculated from $\sigma_{ft}(A=1d,\tau)$, black dots, and Eq. 6, blue diamonds, for simulated FPM noise. Note that the log/log slope for the $\sigma_{ft}(A=1d,\tau)$ data is not constant as compared to the straight fit line (red).

may involve the introduction of a bias term and changes in the value of x as more is learned about the instabilities in time/frequency transfer techniques.

IV. Conclusions

Techniques for characterizing frequency transfer uncertainty have been developed, and much has been learned about the noise levels and noise types of frequency transfer using common-view and carrier-phase GPS, and TWSTFT. The best transfer techniques are TWSTFT and carrier-phase GPS, with frequency transfer uncertainties close to $1x10^{-16}$ at 30 days. However, improved frequency transfer is needed for future frequency standards which may very well have uncertainties in the low 10^{-17} range.

There is still much to be learned about the level and type of noise in frequency transfer. It is not clear what the balance is between WPM and FPM noise and to what extent different transfer techniques are correlated. Is there an annual cycle present and how large is it? To answer many of these questions a third independent transfer method is needed. Unfortunately, there is no immediate prospect for a practical and economical technique, with sufficient stability, to appear in the near future. Two-way time/frequency transfer over optical fibers offers considerable promise, but dedicated fibers covering long (intercontinental) distances are very expensive.

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